INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

May 2000

FURTHER MATHEMATICS

Standard Level

Paper 1

9 pages

Paper 1 Markscheme

Instructions to Examiners

1 All marking must be done using a **red** pen.

2 **Abbreviations**

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown.

C Marks awarded for **Correct** statements

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 **Follow Through (ft) Marks**

Questions in this paper were constructed to enable a candidate to:

- show, step by step, what he or she knows and is able to do;
- use an answer obtained in one part of a question to obtain answers in the later parts of a question.

Thus errors made at any step of the solution can affect all working that follows. Furthermore, errors made early in the solution can affect more steps or parts of the solution than similar errors made later.

To limit the severity of the penalty for errors made at any step of a solution, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise an error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent parts of the question;

(iii) award ***M*** marks for a correct method, and ***A*(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	***M1***	Amount earned = \$ 600 × 1.02	8	**M1**
= \$ 612	***A1***	= \$ 602	×	***A0***
\$ (306 × 1.02) + (306 × 1.04)	***M1***	Amount = 301 × 1.02 + 301 × 1.04	8	***M1***
= \$ 630.36	***A1***	= \$ 620.06	8	***A1*(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by '**(d)**' (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Where alternative methods are included, they often refer to graphic dispaly calculator solutions, and they are indicated by **OR** *e.g.*

Mean = 59 ***(G2)***

OR

Mean = 7820/134 ***(M1)***
= 59 ***(A1)***

Thus, the working out must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by '**(d)**' (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

(a) In the case when the accuracy of answers is **specified in the question** (for example: “all answers should be given to four significant figures”) ***A*** marks are awarded **only if** the correct answers are given to the accuracy required.

(b) When the accuracy is **not** specified in the question, then the general rule applies:

> *Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.*

In this case, the candidate is **penalised once only IN EACH QUESTION** for giving a correct answer to the wrong degree of accuracy. Hence, on the **first** occasion in a question when a correct answer is given to the wrong degree of accuracy ***A*** marks are **not** awarded. But on **all subsequent occasions** in the same question when correct answers are given to the wrong degree of accuracy then ***A*** marks **are** awarded.

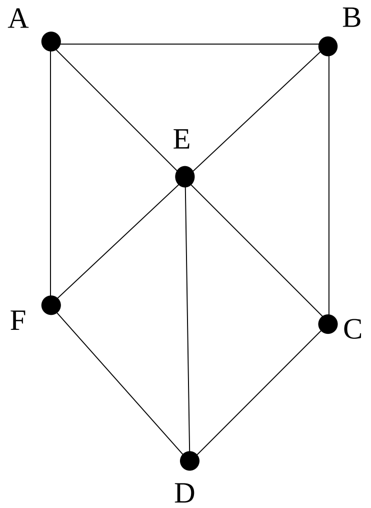
6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. The markschemes will specify where answers only are acceptable, and where candidates are expected to explain what they are doing. The abbreviation ***G*** will be used to indicate marks for answers obtained from a calculator, irrespective of working shown. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded.
Where possible, examples will be provided to guide examiners in awarding these method marks.

1. There are four colours. Since E is connected to all vertices, only A and B can have the same colour as some of the rest. C has to have a different colour from B and F from A, so both colours can still be used, however, D must have a different colour by now. Hence the 4 colours.

Students may use the Chromatic polynomial or any other approach leading to: ***(R1)(R1)***

$\chi(G) = 4$ ***(A1)***



(C2)

Total [5 marks]

2. (a) A set S is well-ordered if every non-empty subset of S has a least element. ***(C1)***
An example is $\mathbb{N}$ ***(A1)***

(b) Let S_1 and S_2 be two well-ordered sets with a non-empty intersection.
$\phi \neq S_1 \cap S_2$ and $S_1 \cap S_2 \subset S_1$. ***(M1)(M1)***
Hence, $S_1 \cap S_2$ has a least element because S_1 is well-ordered. ***(R1)***

Total [5 marks]

3. (a) Let D be the random variable the outside diameter of the tube.
$\mathrm{E}(D) = 3 + 0.3 = 3.3$ inches. ***(A1)***
Standard deviation of $D = \sqrt{\left\{(0.02)^2 + (0.005)^2\right\}} = 0.0206$ inches ***(A1)***

(b) Suppose the probability density function for the random variable X is $f(x)$.
By definition
$\mathrm{Var}(X) = \sum_x \left(x - \mathrm{E}(X)\right)^2 f(x)$ ***(M1)***
$= \sum_x \left\{x^2 f(x) - 2x\mathrm{E}(X) f(x) + \left(\mathrm{E}(X)\right)^2 f(x)\right\}$ ***(M1)***
$= \sum_x x^2 f(x) - 2\mathrm{E}(X)\sum_x xf(x) + \left(\mathrm{E}(X)\right)^2 \sum_x f(x)$ ***(A1)***
$= \mathrm{E}(X^2) - \left(\mathrm{E}(X)\right)^2$ ***(AG)***

Total [5 marks]

4. (a) A series $\sum_{n=1}^{\infty} u_n$ is said to be conditionally convergent, when it is convergent but not absolutely convergent. ***(C1)***

(b) The series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$ is an alternating series where $\frac{1}{\sqrt{n}}$ decreases as n increases.

Also $\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0$. ***(M1)***

By the alternating series test the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$ converges. ***(R1)***

But $\sum_{n=1}^{\infty}\left|(-1)^n \frac{1}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is divergent since $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges for $p \le 1$.

Thus the given series is conditionally convergent. ***(M1)(R1)***

Total [5 marks]

5. Let the set $G = \{e, a, b, a \circ b\}$. Hence $b \circ a$ is one of e, a, b or $a \circ b$. ***(M1)***

If $b \circ a$ is e, then $a \circ b$ is e ***(M1)***

If $b \circ a = a$, then $b = e$ and $a \circ b = a \circ e = a$. ***(M1)***

If $b \circ a = b$, then $a = e$ and consequently $a \circ b = b$. ***(M1)***

If $b \circ a = a \circ b$, then $a \circ b = b \circ a$.

Thus the group $(G, \circ)$ is Abelian. ***(R1)***

Total [5 marks]

6. H_0: 70 % or more will secure at least a passing grade in the mathematics competency test.

H_1: Fewer than 70 % will secure at least a passing grade in the mathematics competency test. ***(M1)***

From the data, we have the following table:

	More than 70 %	Fewer than 70 %	Total
A	75	25	100
B	65	35	100
Total	140	60	200

It is a 2×2 contingency table and hence ν, the number of degrees of freedom

$= (2-1)(2-1) = 1$ ***(A1)***

$\chi^2_{0.95} = 3.84$ (with 1 degree of freedom).

χ^2 computed from the data, with Yates correction, is given by

$$\chi^2 = \frac{(|75-70|-0.5)^2}{70} + \frac{(|65-70|-0.5)^2}{70} + \frac{(|25-30|-0.5)^2}{30} + \frac{(|35-30|-0.5)^2}{30}$$

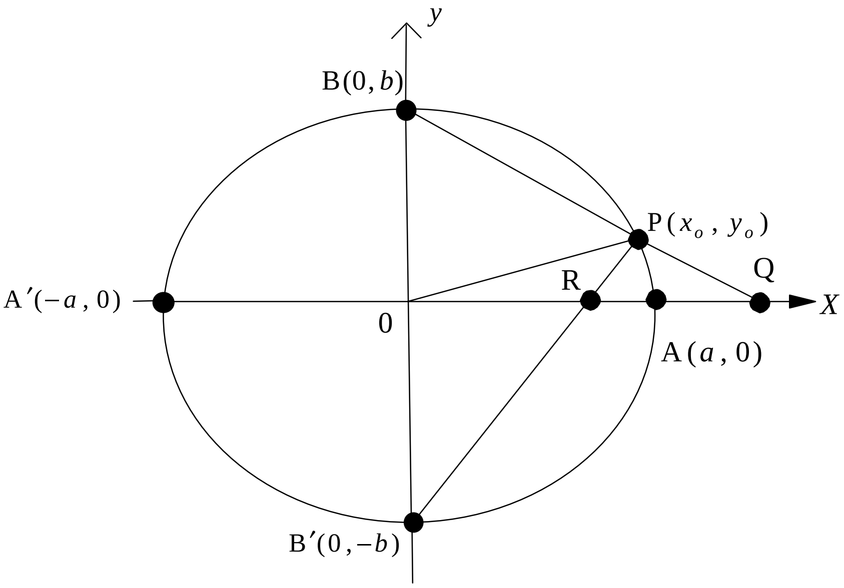
$= 1.93$ ***(M1)(A1)***

Since $1.93 < 3.84$, there is not enough evidence to reject H_0 ***(R1)***

Hence, we accept the claim.

Total [5 marks]

7. Let the centre of the ellipse be the origin and the major and minor axes of the ellipse be the x and y axes, respectively.



The equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the coordinates of the minor axis are $B'(0,-b)$ and $B(0,b)$. ***(M1)*** ***(M1)***

Let the coordinates of the point P be (x_0, y_0).

Equation of line (PB) is $(y-b)x_0=(y_0-b)x$ and the equation of line (PB') is $(y+b)x_0=(y_0+b)x$. ***(M1)***

OQ and OR are the x coordinates of the points of intersection of the lines PB and (PB') with the x axis. Hence, $OQ=\frac{-bx_0}{y_0-b}$ and $OR=\frac{bx_0}{y_0+b}$ ***(A1)***

Thus $OQ.OR=\frac{-x_0^2}{\frac{y_0^2}{b^2}-1}=a^2$, since the point (x_0, y_0) lies on the ellipse ***(R1)***

Total [5 marks]

8. Let $a\equiv b(\bmod m)$. Then $a-b=cm$ for some integer c.

Since, $m<b$, by Euclid's algorithm, there are integers q and r such that $b=qm+r$. ***(M1)***

Now $a=b+cm=qm+r+cm=m(q+c)+r$. ***(M1)***

Hence, when a and b are divided by m, we have the same remainder r. ***(R1)***

Conversely, suppose that $a=qm+r$ and $b=q'm+r$. ***(M1)***

Then $a-b=(q-q')m$ *i.e.* $a\equiv b(\bmod m)$. ***(R1)***

Total [5 marks]

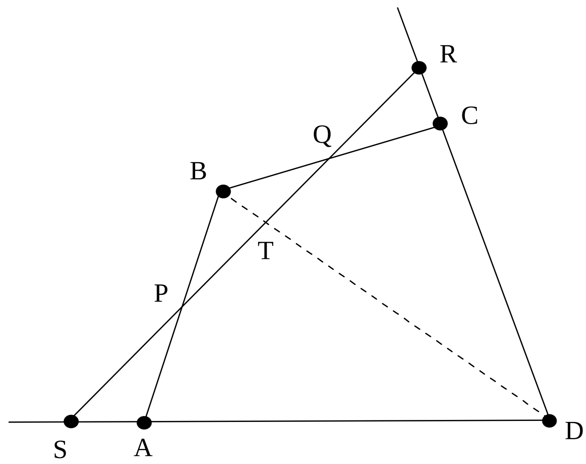
9. Since $\left\{\dfrac{1}{\ln(n+1)}\right\}$ is a decreasing sequence because the denominator gets larger as n gets larger, and since $\lim_{n\to\infty}\dfrac{1}{\ln(n+1)}=0$, the series converges by the alternating series test. ***(M1)*** ***(M1)(AG)***

Since $\left|S-S_n\right|\le\left|a_{n+1}\right|$ in an alternating series and $S_{10}=\dfrac{1}{\ln 2}-\dfrac{1}{\ln 3}\cdots-\dfrac{1}{\ln 11}\approx 0.7197$ the ***(M1)(A1)***

error is less than $\dfrac{1}{\ln 12}\approx 0.402$ (3 s.f.) ***(A1)***

Total [5 marks]

10.



Join the points B to D. Let the intersection point of the line joining points S, P, Q, R with the line (BD) be T. ***(M1)***
By Menalaus' theorem applied to the triangle ΔABD and ΔBCD, we have, ***(M1)***

$$\frac{\overline{DS}}{\overline{SA}}\times\frac{\overline{AP}}{\overline{PB}}\times\frac{\overline{BT}}{\overline{TD}}=-1 \qquad (1)$$

and

$$\frac{\overline{DR}}{\overline{RC}}\times\frac{\overline{CQ}}{\overline{QB}}\times\frac{\overline{BT}}{\overline{TD}}=-1 \qquad (2)$$

(R1)

From (2) $\dfrac{\overline{BT}}{\overline{TD}}=-\dfrac{\overline{RC}}{\overline{DR}}\times\dfrac{\overline{QB}}{\overline{CQ}}$ (3) ***(M1)***

From (1), (2), and (3)

$-\dfrac{\overline{DS}}{\overline{SA}}\times\dfrac{\overline{AP}}{\overline{PB}}\times\dfrac{\overline{QB}}{\overline{CQ}}\times\dfrac{\overline{RC}}{\overline{DR}}=-1$ which shows that ***(AG)***

$$\frac{\overline{DS}}{\overline{SA}}\times\frac{\overline{AP}}{\overline{PB}}\times\frac{\overline{QB}}{\overline{CQ}}\times\frac{\overline{RC}}{\overline{DR}}=1.$$

(R1)

Total [5 marks]